Minitest 2 - Matching

1) Every k-regular bipartite graph $G = (A \cup B, E)$ for $k \geq 1$ has a matching of size $|A|$.

- [x] True
- [ ] False

Corollary : Every k-regular bipartite G has a perfect matching

2) The (cardinality-)maximum matching in a bipartite graph can be found in time $O(\sqrt{|V|}|E|)$.

- [x] True
- [ ] False

für bipartite Graphen

$O(|V|^{1/2} \cdot |E|)$ Hopcroft-Karp (ungewichtet)

$\sqrt{|V|}$

4) If G is a bipartite graph and M is matching in G which is *not cardinality-maximal*, then there is an augmenting path in G with respect to M.

- [x] True
- [ ] False

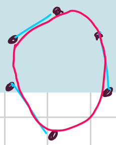
Matching
Berge's Theorem

- Augmenting Path : "path with edges not in M, in M, ... , not in M "
  - An augmenting path is an alternating path that starts from and ends on unmatched/not covered vertices
  - Alternating Path :
    - An alternating path is a path that begins with an unmatched/not covered vertex whose edges belong alternately to the matching and not to the matching

A Matching M is (cardinality-) maximum $\iff$ There's no M-augmenting path

6) For a complete graph G with even number of vertices and positive weights $\ell(x, y)$ satisfying the triangle inequality, let M be the minimum cost perfect matching, and C the minimum cost Traveling Salesman Tour. Then $\ell(M) \leq \ell(C)/2$.

- [x] True
- [ ] False



See 2 Approximation TSP

9) There is a polynomial time algorithm finding 1.5-approximation to the Metric Traveling Salesman Problem.

- [x] True
- [ ] False

See 1.5 Approximation TSP

Minitest 2 - Coloring

3) Every graph without a triangle has chromatic number at most 100.

- ○ True
- ☒ False

Being triangle-free does not imply any fixed upper-bound for $\chi(G)$

5) If G is a connected graph of maximum degree 100, then G has proper coloring into 100 colors, unless G is a complete graph.

- ☒ True
- ○ False

not complete, cannot be a cycle

Brook's Theorem

- $G \neq K_n$, $G \neq C_{2n+1}$, G connected
  - can be colored in O(|E|) time with $\Delta(G)$ colors

7) If G is a graph with maximum degree $\Delta(\mathrm{G})$, a greedy algorithm always finds proper coloring into at most $\Delta(\mathrm{G}) + 1$ colors.

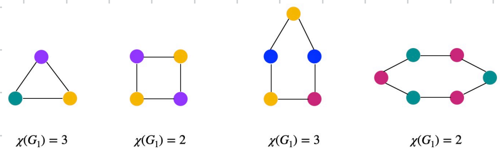
- ☒ True
- ○ False

- All Graphs
  - can be colored in O(|E|) time with $\Delta(G) + 1$ colors

8) Every cycle has a proper 2-coloring.

- ○ True
- ☒ False

odd cycles only possible w. 3 colors



$\chi(C_{2n+1}) = 3$ $\chi(C_{2n}) = 2$

10) There is an efficient algorithm which finds a proper coloring into 6 colors for any planar graph.

- ☒ True
- ○ False

- For planar graphs heuristic+greedy finds a coloring with $\leq$ 6 colors