Minitest 1

1) Every graph has an even number of odd degree vertices.

- [x] Wahr
- [ ] Falsch

2) Every articulation point in a graph is incident to a bridge.

- [ ] Wahr
- [x] Falsch

3) There is a 5-connected graph with all vertices of degree exactly 3.

- [ ] Wahr
- [x] Falsch

4) A Hamiltonian cycle in a graph visits every vertex exactly once. ★

- [x] Wahr
- [ ] Falsch

5) For two edges a, b of a graph, the relation a ~ b when a and b are on a common cycle is an equivalence relation.

- [x] Wahr
- [ ] Falsch

6) Every connected graph with all vertices of even degree has an Eulerian cycle.

- [x] Wahr
- [ ] Falsch

7) Every 2-connected graph has a Hamiltonian cycle.

- [ ] Wahr
- [x] Falsch

8) If $G = (V, E)$ is a 3-connected graph and $v \in V$, then $G[V \setminus \{v\}]$ is 2-connected.

- [x] Wahr
- [ ] Falsch

9) For every $t \geq 3$ a complete graph K_t on t vertices is 2-connected.

- [x] Wahr
- [ ] Falsch

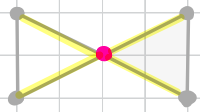
Explanation:

Handshaking lemma: $\sum_{v \in V} \deg(v) = 2 \cdot |E|$

odd · odd = odd
odd · even = even

even + even = even
odd + even = odd

Counterexample:



Articulation point

: not a bridge

(Vertex-) Connectivity ≤ Edge-Connectivity ≤ minimum degree

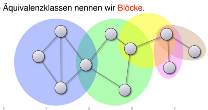
Def of hamiltonian cycle

Def of equivalence relation

Definition: Sei $G = (V, E)$. Wir definieren eine Äquivalenzrelation auf E durch

$$e \sim f :\Longleftrightarrow \begin{cases} e = f, & \text{oder} \\ \exists \text{ Kreis durch } e \text{ und } f \end{cases}$$

Die Äquivalenzklassen nennen wir Blöcke.

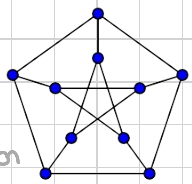


Satz: Ein zusammenhängender Graph G = (V, E) enthält eine Eulertour gdw. der Grad jedes Knotens gerade ist.
… und eine solche kann man in O(|E|) Zeit finden

Euler's theorem:

G has an Eulerian Cycle ⟺ Every vertex has an even degree and G is connected

Counterex: Petersen G



3-connected (thus certainly 2 connected)

does NOT have a hamiltonion cycle.

Def of k-connectivity

3-connected: you have to remove at least 3 vertices to make G disconnected

2-connected: " at least 2 "

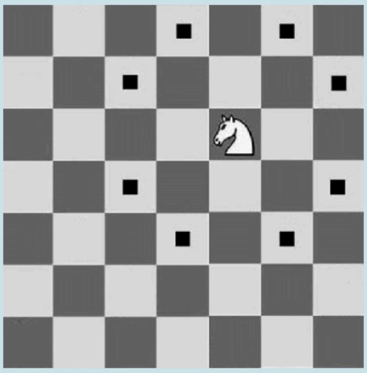
Def of K_t: Every pair of distinct vertices is directly joined by an edge

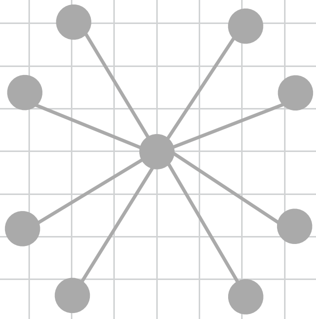
remove 1 vertex from K_t

resulting graph is K_{t-1}

10) There is a Hamiltonian cycle for a chess knight jumping on 7×7 chessboard.

As a reminder, legal chess knight moves are shown below.





hamiltonian cycle ×

- [ ] Wahr
- [x] Falsch