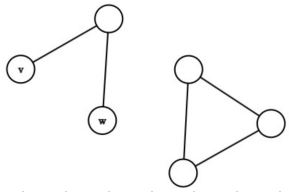
# Quiz 9

1)




True or false: In the graph above, the nodes labeled $v$ and $w$ are adjacent.

Select one:

$\{v, w\} \notin E$

True

False

2)

Let $G = (V, E)$ be a graph with 4 vertices $v_1, v_2, v_3, v_4$. Suppose that

$$\deg(v_1) = 1,\ \deg(v_2) = 2,\ \deg(v_3) = 3,\ \deg(v_4) = 2.$$

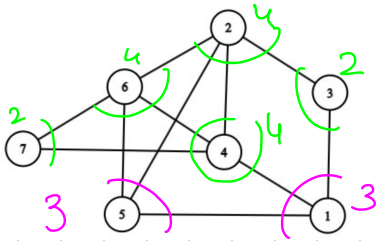
How many edges does $G$ have? *(Your answer should consist of a single integer.)*

$1+2+3+2 = 8 = 2|E|$

$|E| = 4$

Answer: 4

3)



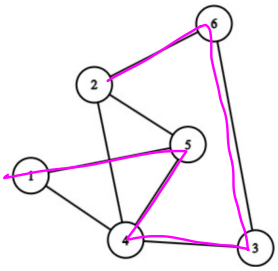

True or false: the graph above has a closed Eulerian walk (Eulerzyklus).

Select one:

True

False

4)




True or false: the graph above has a Hamiltonian path (Hamiltonpfad).

Select one:

True

False

5)

Let $G = (V, E)$ and assume *all vertices of $G$ have even degree*. Recall the algorithm walk from the lecture:

```
walk(u):
    if there exists an edge {u, v} which is not marked:
        mark the edge {u, v}
        walk(v)
```

for sure ✓

Let $u \in V$ be a vertex of $G$. Which of the following statements must be true after executing walk$(u)$?

*(Below, an edge is called* incident *to $u$ if it is of the form $\{u, v\}$, where $v$ is another vertex in $G$)*

Select one or more:

both could be true

#marked edges: 3 4

a. The total number of marked edges in $G$ is even.

b. The total number of marked edges in $G$ is odd.

c. ✓ The total number of unmarked edges *incident to* $u$ is even.

d. ✗ The total number of unmarked edges *incident to* $u$ is odd.

for sure (fact from lecture)

## What is used?

### Graph
**adjacent / incident**

- $v_i$ and $v_j$ are adjacent iff : $\{v_i, v_j\} \in E \quad (v_i, v_j) \in E$
- $v_i$ and $e_k$ are incident iff : $v_i \in e_k$

### Graph
**Handshaking Lemma**

$$\sum_{v \in V} \deg(v) = 2|E| = 2m$$

### Graph
**Eulerian Lemmas**

- A **Eulerian walk** (german "Eulerweg") is a walk that contains every edge exactly once.
- A **closed Eulerian walk** (german "Eulerzyklus") is a closed walk that contains every edge exactly once.

∃ Eulerian Walk ⟺ <= 2 vertices have odd degree (start and end)

Ausser für Start- und Endknoten gilt: #hin = #weg

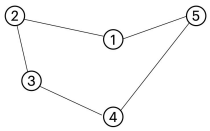
∃ Eulerian Closed Walk ⟺ G is connected and every vertex has even degree

### Graph
**Hamiltonian Path / Hamiltonian Cycle**

- Hamiltonian Path: A **Hamiltonian path** (german "Hamiltonpfad") is a path that contains every vertex.
- Hamiltonian Cycle: A **Hamiltonian cycle** (german "Hamiltonkreis") is a cycle that contains every vertex.

Is it a hamiltonian path? Is it a hamiltonian cycle?

| | | |
|---|---|---|
| (5, 1, 2, 3, 4) | ✓ | ✗ |
| (5, 1, 2, 3, 4, 5) | ✗ | ✓ |




Walk(u): (finde möglichst langen Weg von u ohne wiederholte Kanten)

if $\exists v$. $uv \in E$, nicht markiert

markiere $uv$

Walk(v)

Im Beispiel: keine Kanten markiert zu anfangs

Walk(B): B A C B D E F B

Walk(E): E G H E

am Ende sind alle Kanten markiert

Eigenschaften

1. Walk(u) markiert Weg W mit Startknoten u
2. jede Kante höchstens einmal markiert
3. Endknoten von W hat alle Kanten markiert

Invariante: $\forall v \in V$. Anzahl unmarkierter Kanten an v gerade