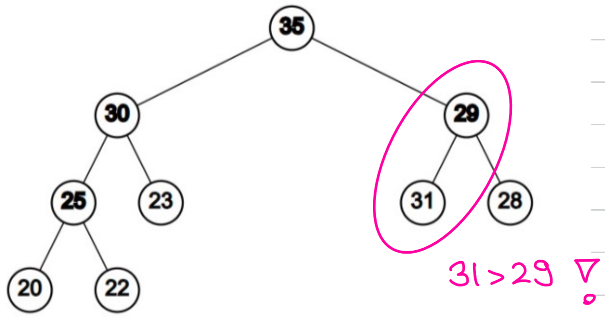
Quiz 6

1)



31 > 29 !

True or false: *The binary tree above satisfies the max heap condition (for every node).*

Select one:

True

False

what is used ?

Max Heap Condition:

$\mathrm{val}(n) \geq \mathrm{val}(\mathrm{children}(n))$

2)

Let T be a (max) heap. Let v be a node at depth 2 in T, and let w be a node at depth 4 in T.

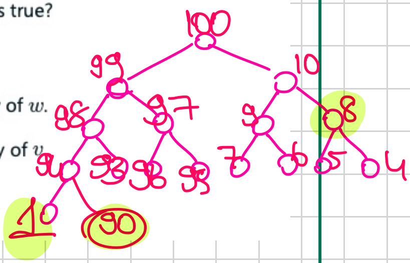
Which of the following statements about the keys of v and w is true?

Select one:

a. The key of v **must be larger than or equal to** the key of w.

b. The key of w **must be larger than or equal to** the key of v.

c. None of the above.



Heap Knowledge

3)

In the lecture you saw the ExtractMax procedure to remove the root of a (max) heap, and subsequently restore the heap condition.

Let ℓ be the rightmost leaf in the lowest level of a max heap consisting of at least 2 nodes, and assume all keys in the heap are unique.

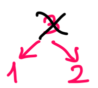
True or false: *The node ℓ is guaranteed to be a leaf again after running the ExtractMax procedure.*

(A leaf is a node without children)

Select one:

True

False



2
↓
1 → not a leaf!

4)

True or false: *the runtime of the quick sort algorithm depends on the way in which the pivot is chosen.*

Select one:

True

False

$O(n \log n)$ if pivot divides the array evenly

$O(n^2)$ if pivot is the largest/smallest element

5)

Let B be the binary decision tree of a comparison-based sorting algorithm for arrays of length n.

Which of the following statements are true for B?

Select one or more:

~~a. B contains at most 2^n nodes.~~ → $2^{h+1} \geq$ #nodes

✓ b. B contains at least $n!$ nodes.

✓ c. B has depth at least $\Omega(n \log n)$.

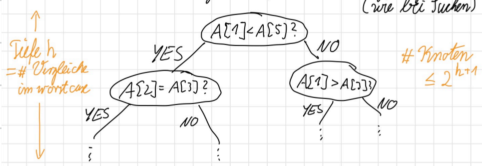
~~d. B has depth at most $O(n \log n)$.~~

there's no limit of how bad your algo can be!

GEHT ES BESSER? — aber kompliziert — hin und her

Antwort: NEIN (Annahme: vergleichsbasiertes Sortieren)

Beweis: Betrachte Entscheidungsbaum für einen beliebigen Algorithmus (wie bei Suchen)



Es muss gelten: # Knoten ≥ # mögliche Output = $n!$

Umordnungen von A, z.B. | A(1) | A(2) | A(3) | ... | A(n) |, | A(2) | A(1) | A(3) | ... | A(n) |, | A(3) | A(1) | A(2) | ... | A(n) |, ...

$\rightsquigarrow 2^{h+1} \geq$ # Knoten $\geq n!$

$\rightsquigarrow h \geq \log_2(n!) - 1 \geq \Omega(n \log n)$