Quiz 2

what is used ?

1) You want to show using induction that a statement $A(n)$ holds for all n of the form $n = 2^k$, with $k \in \mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$

Which of the following combinations of *base case* and *induction step* would form a valid proof?

Select one or more:

a. *Base case:* $A(1)$ holds.
Induction step: $A(n) \implies A(2n)$ for all integers $n \geq 1$. ✓ still true !

b. *Base case:* $A(0)$ holds.
Induction step: $A(2^k) \implies A(2^{k+1})$ for all integers $k \geq 1$.

c. *Base case:* $A(1)$ holds.
Induction step: $A(2^k) \implies A(2^{k+1})$ for all integers $k \geq 1$.

d. *Base case:* $A(1)$ holds.
Induction step: $A(2^k) \implies A(2^{k+1})$ for all integers $k \geq 0$. ✓ more convenient

Induction Explanation W1

2) $\sum_{k=1}^{n} k^{0.3} \leq O(n^{1.3})$?

$$\sum_{k=1}^{n} k^{0.3} = \Theta(n^{1.3}) \Rightarrow \leq O(n^{1.3})$$

(detailed: $\sum_{k=1}^{n} k^{0.3} \leq \sum_{k=1}^{n} n^{0.3} = n \cdot n^{0.3} = n^{1.3}$)

$$\sum_{i=1}^{n} i^k = \Theta(n^{k+1})$$

3) Recall that $f = \Theta(g)$ if and only if $f \leq O(g)$ and $g \leq O(f)$. → given

Is it true that $n^2 + n - 1 = \Theta(n^3 - n^2)$?

$$n^2+n-1 \leq O(n^3-n^2) \text{ but } n^3-n^2 \not\leq O(n^2+n-1)$$

or $$\lim_{n\to\infty} \frac{n^2+n-1}{n^3-n^2} = 0 \neq C \in \mathbb{R}^+$$

alternative :

- *If* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C \in \mathbb{R}^+$, *then* $f = \Theta(g)$.

4) Consider the following pseudocode snippet:

```
i ← 1
while i ≤ n:
    i ← i + 2
    f()
```

Which of the following expressions in Θ-notation correctly describe the number of calls to f?

Select one or more:

a. $\Theta(n^2)$

b. $\Theta(n \log n)$

✓ c. $\Theta(n)$

✓ d. $\Theta(n/2)$ $= \Theta(n)$ /2 doesn't make a difference

$$\sum_{i=1}^{n/2} 1 = n/2 = \Theta(n)$$

Loop Counting Explanation W3

5) Consider the following pseudocode snippet:

```
x ← n^2
while x ≥ 2:
    x ← x/2        (x is not necessarily an integer!)
    f()
```

x : $n^2, \frac{n^2}{2}, \frac{n^2}{2^2}, \frac{n^2}{2^3} \ldots$

Assume $n \geq 2$. Which of the following expressions in Θ-notation correctly describes the number of calls to f?

$n = 2^k$ $\log_2(n^2)$ times

a. $\Theta(\log_2 n)$

b. $\Theta(n^2 \log_2 n)$

c. $\Theta(n/2)$

d. $\Theta((\log_2 n)^2)$

$$\log_2(n^2) = 2 \cdot \log_2(n) = \Theta(\log_2 n)$$

Loop Counting Explanation W3