

Quiz 2

1) You want to show using induction that a statement $A(n)$ holds for all n of the form $n = 2^k$, with $k \in \mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$. Which of the following combinations of *base case* and *induction step* would form a valid proof?

Select one or more:

- a. *Base case:* $A(1)$ holds.
Induction step: $A(n) \implies A(2n)$ for all integers $n \geq 1$.
- b. *Base case:* $A(0)$ holds.
Induction step: $A(2^k) \implies A(2^{k+1})$ for all integers $k \geq 1$.
- c. *Base case:* $A(1)$ holds.
Induction step: $A(2^k) \implies A(2^{k+1})$ for all integers $k \geq 1$.
- d. *Base case:* $A(1)$ holds.
Induction step: $A(2^k) \implies A(2^{k+1})$ for all integers $k \geq 0$.

2) $\sum_{k=1}^n k^{0.3} \leq O(n^{1.3})$?

3) Recall that $f = \Theta(g)$ if and only if $f \leq O(g)$ and $g \leq O(f)$.
Is it true that $n^2 + n - 1 = \Theta(n^3 - n^2)$?

what is used?

Induction Explanation W1

$$\sum_{i=1}^n i^k = \Theta(n^{k+1})$$

→ given

alternative:

• If $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = C \in \mathbb{R}^+$, then $f = \Theta(g)$.

4) Consider the following pseudocode snippet:

```
i ← 1
while i ≤ n:
  i ← i + 2
  f()
```

Which of the following expressions in Θ -notation correctly describe the number of calls to f ?

Select one or more:

- a. $\Theta(n^2)$
- b. $\Theta(n \log n)$
- c. $\Theta(n)$
- d. $\Theta(n/2)$

Loop Counting Explanation W3

5) Consider the following pseudocode snippet:

```
x ← n2
while x ≥ 2:
  x ← x/2      (x is not necessarily an integer!)
  f()
```

Assume $n \geq 2$. Which of the following expressions in Θ -notation correctly describes the number of calls to f ?

- a. $\Theta(\log_2 n)$
- b. $\Theta(n^2 \log_2 n)$
- c. $\Theta(n/2)$
- d. $\Theta((\log_2 n)^2)$

Loop Counting Explanation W3