Quiz 1

1) $n^6 + 10n^5 + 20n^2 = O(n^6)$

2) $n^4 - 5n^2 \leq O(\frac{n^4}{\log n})$.

3) Which of these statements is correct?

1. $e^{3\log(n)} \leq O(n)$
2. $e^{\log(3)+\log(n)} \leq O(n)$

4) Let $f : \mathbb{N} \to \mathbb{R}$, $g : \mathbb{N} \to \mathbb{R}$ and $h : \mathbb{N} \to \mathbb{R}$ be three functions.
Assume that $f(n) \leq O(n \cdot g(n))$ and $g(n) \leq O(n^2 \cdot h(n))$.
Then it must be true that $f(n) \leq O(n^3 h(n))$.

5) We want to prove that $2^{2n} \leq (2n)!$ for all integers $n \geq 2$ using induction.
Which of the following is a correct *base case* for the inductive argument?

a. $2^2 = 4 \leq 24 = 4! = (2 \cdot 2)!$

b. $2^{2 \cdot 2} = 2^4 = 16 \leq 24 = 4! = (2 \cdot 2)!$

c. For some k, assume that $2^{2k} \leq (2k)!$

d. $2^{2 \cdot 3} = 2^6 = 64 \leq 720 = 6! = (2 \cdot 3)!$

what is used ?

Theorem 1. *Let N be an infinite subset of $\mathbb{N}$ and $f : N \to \mathbb{R}^+$ and $g : N \to \mathbb{R}^+$.*

- *If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$, then $f \leq O(g)$ and $g \not\leq O(f)$.*
- *If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C \in \mathbb{R}^+$, then $f \leq O(g)$ and $g \leq O(f)$.*
- *If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, then $f \not\leq O(g)$ and $g \leq O(f)$.*

$$b^x = (e^{\ln b})^x = e^{(\ln b)\cdot x}.$$

$$\log_a(bc) = \log_a(b) + \log_a(c)$$
$$\log_a(b^c) = c\log_a(b)$$

Definition 1 (O-Notation). For $f : N \to \mathbb{R}^+$,

$$O(f) := \{g : N \to \mathbb{R}^+ \mid \exists C > 0\, \forall n \in N\ g(n) \leq C \cdot f(n)\}.$$

Induction: base case explanation from last week