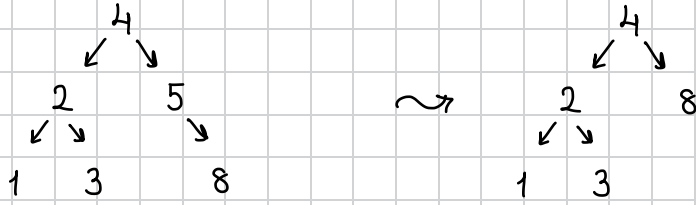


# BST - remove(x)

Case 1: remove(1)



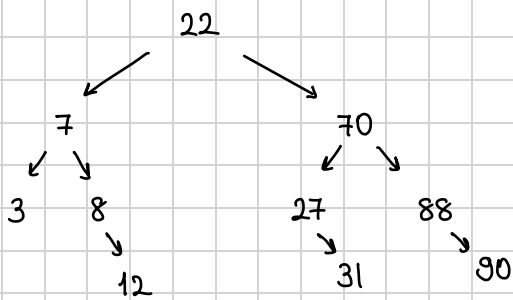
Case 2: remove(5)



Case 3: remove(2) / bigger BST



remove(22):

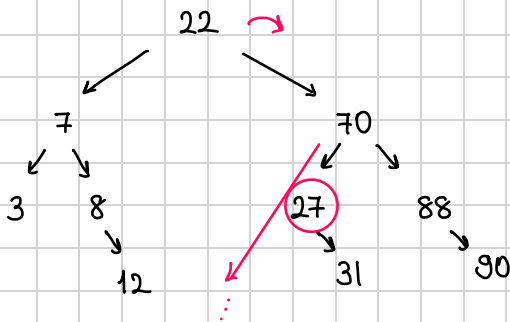


★ Find the next biggest element!

How?

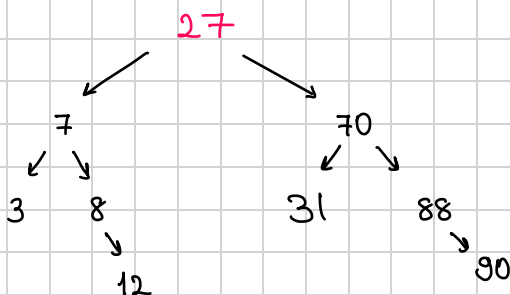
walk once to the right and all the way to the left

Or: find the previous largest element walk once to the left and all the way to the right



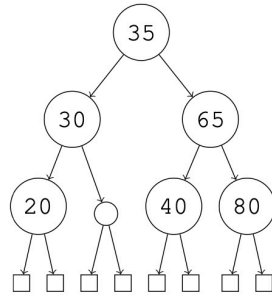
Note: will be case 1 or 2

we "remove(27)" and put it in the place of 22.

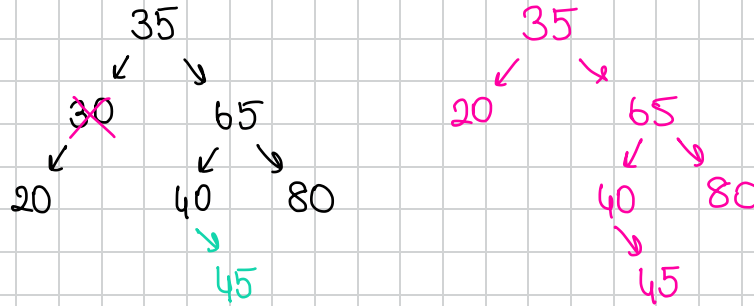
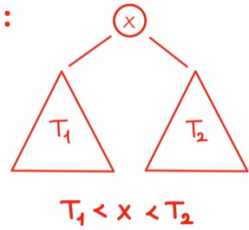


# BST + AVL - exam questions

i) Draw the **binary search tree** obtained from the following tree by performing the two operations INSERT(45) and DELETE(30), in that order.

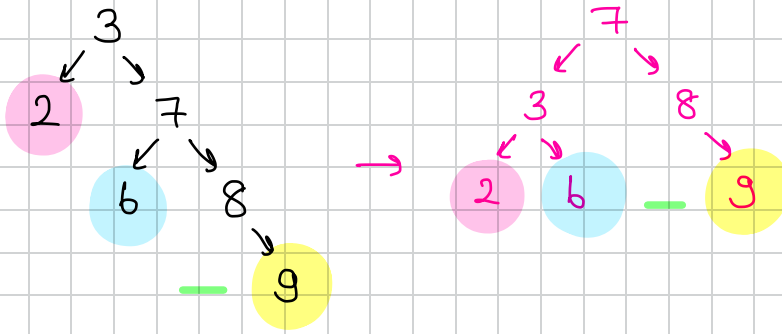
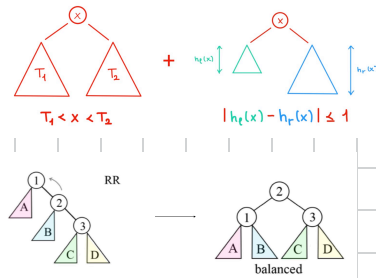


BST Condition :

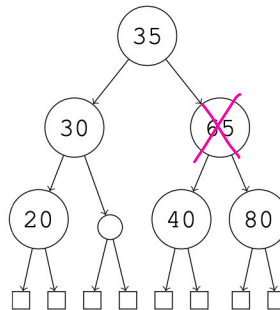


ii) Draw the **AVL tree** that is obtained when inserting the keys 3, 2, 7, 6, 8, 9 in this order into an empty tree (it suffices to draw only the final tree).

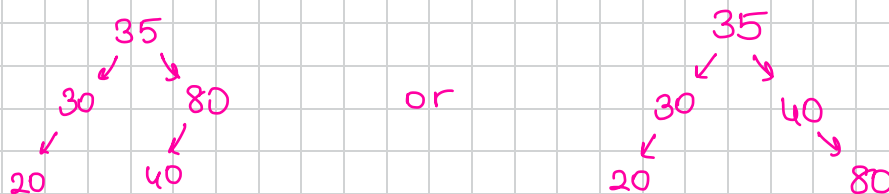
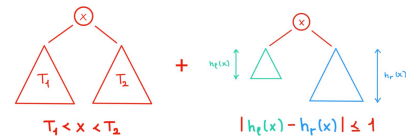
AVL-Tree Condition :



iii) Draw the **AVL tree** that is obtained by deleting key 65 from the tree below.



AVL-Tree Condition :



# DP - Introduction

Consider the recurrence

$$A_1 = 1$$

$$A_2 = 2$$

$$A_3 = 3$$

$$A_4 = 4$$

$$A_n = A_{n-1} + A_{n-3} + 2A_{n-4} \text{ for } n \geq 5.$$

Compute  $A_n$  using bottom-up dynamic programming and state the run time of your algorithm. Address the following aspects in your solution:

- (1) **Definition of the DP table:** What are the dimensions of the table  $DP[\dots]$ ? What is the meaning of each entry?
- (2) **Computation of an entry:** How can an entry be computed from the values of other entries? Specify the base cases, i.e., the entries that do not depend on others.
- (3) **Calculation order:** In which order can entries be computed so that values needed for each entry have been determined in previous steps?
- (4) **Extracting the solution:** How can the final solution be extracted once the table has been filled?
- (5) **Run time:** What is the run time of your solution?

0) Dimensions of the DP table:  $DP[1 \dots n]$

1) Definition of the DP table:  $DP[i] = A_i$  for  $1 \leq i \leq n$

2) Computation of an entry:

Initialization:  $DP[1] = 1, DP[2] = 2, DP[3] = 3, DP[4] = 4$

Recursion:  $DP[k] = DP[k-1] + DP[k-3] + 2 \cdot DP[k-4]$   
for  $k \geq 5$

3) Calculation order:

We calculate with increasing  $i$

4) Extracting Solution: The result is at:  $DP[n]$

5) Runtime: Each entry can be computed in:  $\Theta(1)$

We're filling a DP table of size:  $n$

So the runtime is  $\Theta(n)$