Mini-exam (Induction + Asymptotic Notation)

/ 5 P

a) *Asymptotic notation quiz:* For each of the following claims, state whether it is true or false. You get 1P for a correct answer, -1P for a wrong answer, 0P for a missing answer. You get at least 0 points in total.

Assume $n \geq 4$.

Claim	true	false
$n^3 + n^4 = \Theta(n^4)$	☐	☐
$n^{10} \leq O(\log(n)^{100})$	☐	☐
$1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \leq O(2^n)$	☐	☐
Suppose $a_1 = 1$ and $a_{i+1} = 3a_i + 1$ for all $i \geq 2$. Then $a_n \leq O(4^n)$.	☐	☐
$2^{10\log(n)} = \Theta(2^{20\log(n)})$	☐	☐

/ 4 P

b) *Induction:* Consider the Fibonacci numbers $(F_n)_{n\in\mathbb{N}}$, which are given by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$. Show by mathematical induction that for any integer $n \geq 0$,

$$F_{n+1}^2 \geq \sum_{k=0}^{n} F_k^2.$$

Hint: *Use the facts that $F_{n+1} \geq F_n$ and $F_n \geq 0$ for all $n \in \mathbb{N}$ (you don't need to justify that).*