Loop Counting Examples

Exercise 3.3 Counting function calls in loops (1 point).

For each of the following code snippets, compute the number of calls to f as a function of $n \in \mathbb{N}$. Provide **both** the exact number of calls and a maximally simplified asymptotic bound in Θ notation.

Algorithm 1 (a) i $\leftarrow 0$ while $i \le n$ do f() j $\leftarrow 0$ while $j \le 2n$ do f() j $\leftarrow j + 1$																				
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Theory Task T2.

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In this part, you should justify your answers briefly.

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a) Counting iterations: For the following code snippets, derive an asymptotic bound for the number of times f is called. Simplify the expression as much as possible and state it in Θ -notation as concisely as possible.

i) Snippet 1:

Algorithm 1		
for $i = 1, \ldots, n$ do		
for $j = 1, \ldots, i^2$ do		
f()		
f()		

ii) Snippet 2:

Algorithm 2		
for $i = 1, \ldots, n$ do		
$k \leftarrow 1$		
while $k \leq i^2 \operatorname{do}$		
f()		
$k \leftarrow 2k$		
f()		