Loop Counting Examples

Exercise 3.3 *Counting function calls in loops* **(1 point)**.

For each of the following code snippets, compute the number of calls to f as a function of $n \in \mathbb{N}$. Provide **both** the exact number of calls and a maximally simplified asymptotic bound in Θ notation.

Algorithm 1

(a)
```
i ← 0
while i ≤ n do
    f()
    f()
    i ← i + 1
j ← 0
while j ≤ 2n do
    f()
    j ← j + 1
```

Algorithm 2

(b)
```
i ← 1
while i ≤ n do
    j ← 1
    while j ≤ i^3 do
        f()
        j ← j + 1
    i ← i + 1
```

for the mini cheat sheet:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$$

Theory Task T2. **/ 15 P**

In this part, you should **justify your answers briefly**.

/ 4 P a) *Counting iterations:* For the following code snippets, derive an asymptotic bound for the number of times f is called. Simplify the expression as much as possible and state it in Θ-notation as concisely as possible.

i) Snippet 1:

Algorithm 1

```
for i = 1, ..., n do
    for j = 1, ..., i^2 do
        f()
    f()
```

ii) Snippet 2:

Algorithm 2

```
for i = 1, ..., n do
    k ← 1
    while k ≤ i^2 do
        f()
        k ← 2k
    f()
```