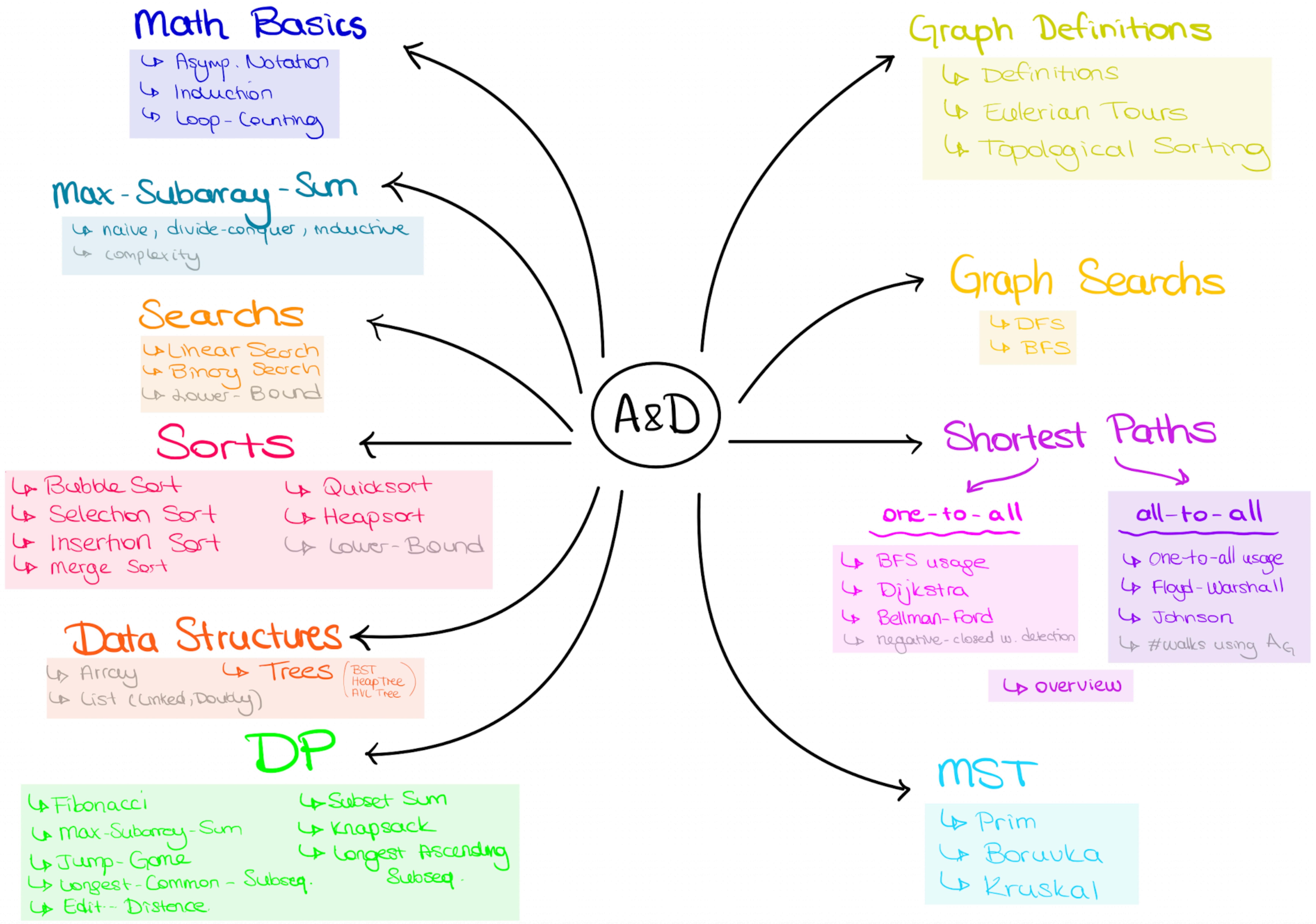
A&D

Exercise Session 2

Nil Ozer

A&D Overview



A&D
Math Basics
Asymp. Notation
Induction
Loop-Counting
Max-Subarray-Sum
naive, divide-conquer, inductive
complexity
Searchs
Linear Search
Binary Search
Lower-Bound
Sorts
Bubble Sort
Selection Sort
Insertion Sort
Merge Sort
Quicksort
Heapsort
Lower-Bound
Data Structures
Array
List (Linked, Doubly)
Trees
BST
Heap Tree
AVL Tree
DP
Fibonacci
Max-Subarray-Sum
Jump-Game
Longest-Common-Subseq.
Edit-Distance
Subset Sum
Knapsack
Longest Ascending Subseq.
Graph Definitions
Definitions
Eulerian Tours
Topological Sorting
Graph Searchs
DFS
BFS
Shortest Paths
one-to-all
BFS usage
Dijkstra
Bellman-Ford
negative-closed w. detection
all-to-all
One-to-all usage
Floyd-Warshall
Johnson
#walks using A_G
overview
MST
Prim
Boruvka
Kruskal

Outline

- Quick recap
- Quiz
- Asymptotic Notation
- Exercise Sheet 1
- Mini-exam

Quick recap

$\lim_{n \to \infty}$: $1 < \log(\log(n)) < \log(n) < \sqrt{n} < n < n \cdot \log(n) < n \cdot \sqrt{n} < n^2 < 2^n < n! < n^n$

$n^x < x^n$ (x being fixed)

Log- Rules:

$\log_a(bc)$	$=$	$\log_a(b) + \log_a(c)$
$\log_a(b^c)$	$=$	$c\log_a(b)$
$\log_a(1/b)$	$=$	$-\log_a(b)$
$\log_a(1)$	$=$	0
$\log_a(a)$	$=$	1
$\log_a(a^r)$	$=$	r
$\log_{1/a}(b)$	$=$	$-\log_a(b)$
$\log_a(b)\log_b(c)$	$=$	$\log_a(c)$
$\log_b(a)$	$=$	$\frac{1}{\log_a(b)}$
$\log_{a^m}(a^n)$	$=$	$\frac{n}{m}, \quad m \neq 0$

$$\log_b a = \frac{\log_d a}{\log_d b}$$

Definition 1 (O-Notation). For $f : N \to \mathbb{R}^+$,

$$O(f) := \{g : N \to \mathbb{R}^+ \mid \exists C > 0\ \forall n \in N\ g(n) \leq C \cdot f(n)\}.$$

Theorem 1. *Let N be an infinite subset of $\mathbb{N}$ and $f : N \to \mathbb{R}^+$ and $g : N \to \mathbb{R}^+$.*

- *If* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$, *then* $f \leq O(g)$ *and* $g \nleq O(f)$.
- *If* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C \in \mathbb{R}^+$, *then* $f \leq O(g)$ *and* $g \leq O(f)$.
- *If* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, *then* $f \nleq O(g)$ *and* $g \leq O(f)$.

▸ **Converting exponentials from base b to base e**

Q3: How do we convert b^x to $e^{(\text{something})}$?

A3: Using $\boldsymbol{b = e^{\ln b}}$ we have the **conversion formula**: $b^x = (e^{\ln b})^x = e^{(\ln b)\cdot x}$.

Example 3 Rewrite $\sqrt[3]{7}$ as an exponential with base e.

$x^{100} = (e^{\ln(x)})^{100} = e^{\ln(x) \cdot 100}$

$$\sqrt[3]{7} = 7^{\frac{1}{3}} = \left(e^{\ln 7}\right)^{\frac{1}{3}} = e^{\frac{1}{3}\ln 7}$$

Quiz

Asymptotic Notation

Asymptotic Notation

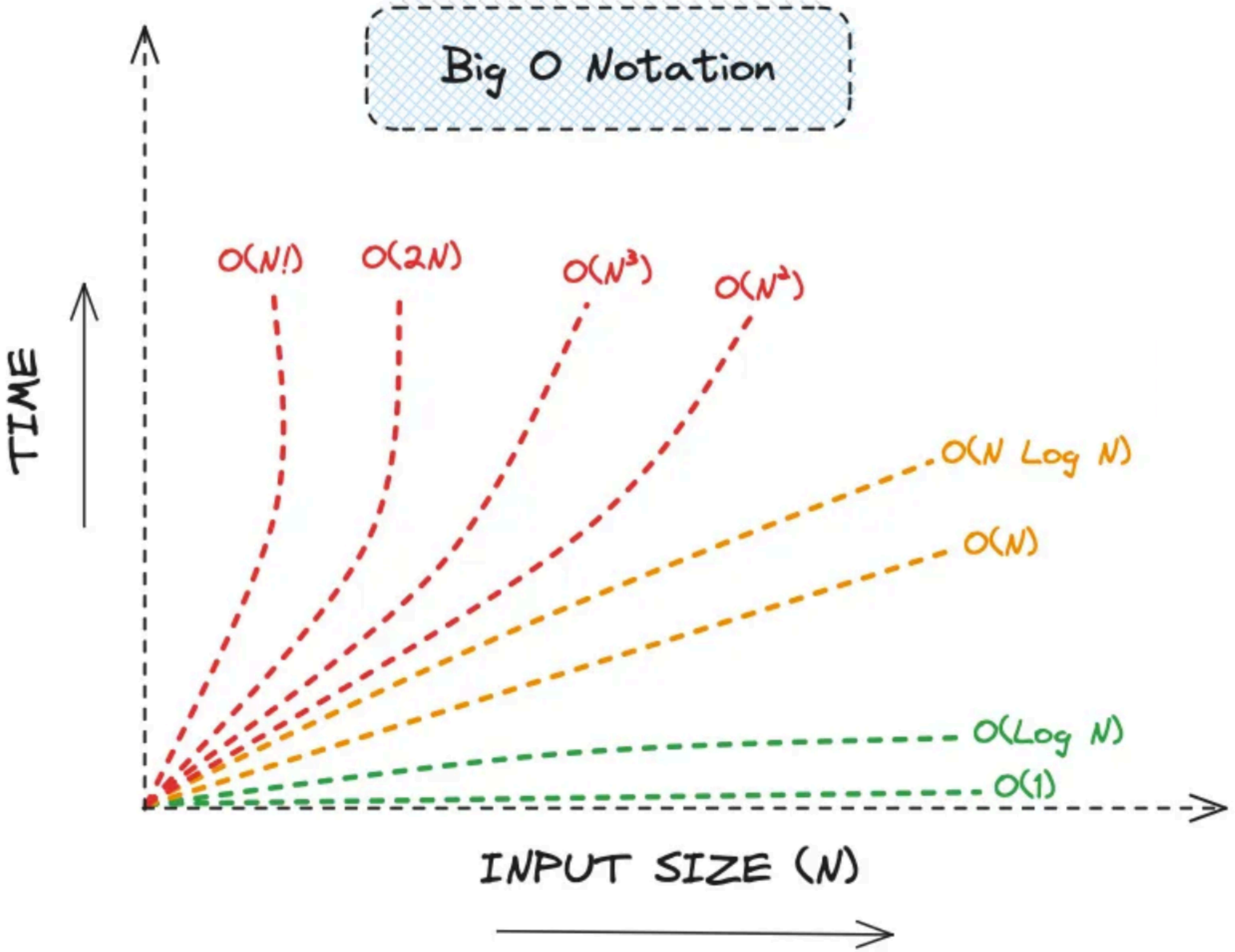
O-Notation

- Describes the upper-bound of an algorithm
- To express the worst-case running time
- Describes the performance of an algorithm as the amount of data increases
- As the input size of the algorithm grows, the execution time of the algorithm is also going to grow. (Constant, linear, logarithmic, exponential...)

- n represents the amount of data that we're passing in
- We don't care about the differences, when those differences are constant values. We care about n

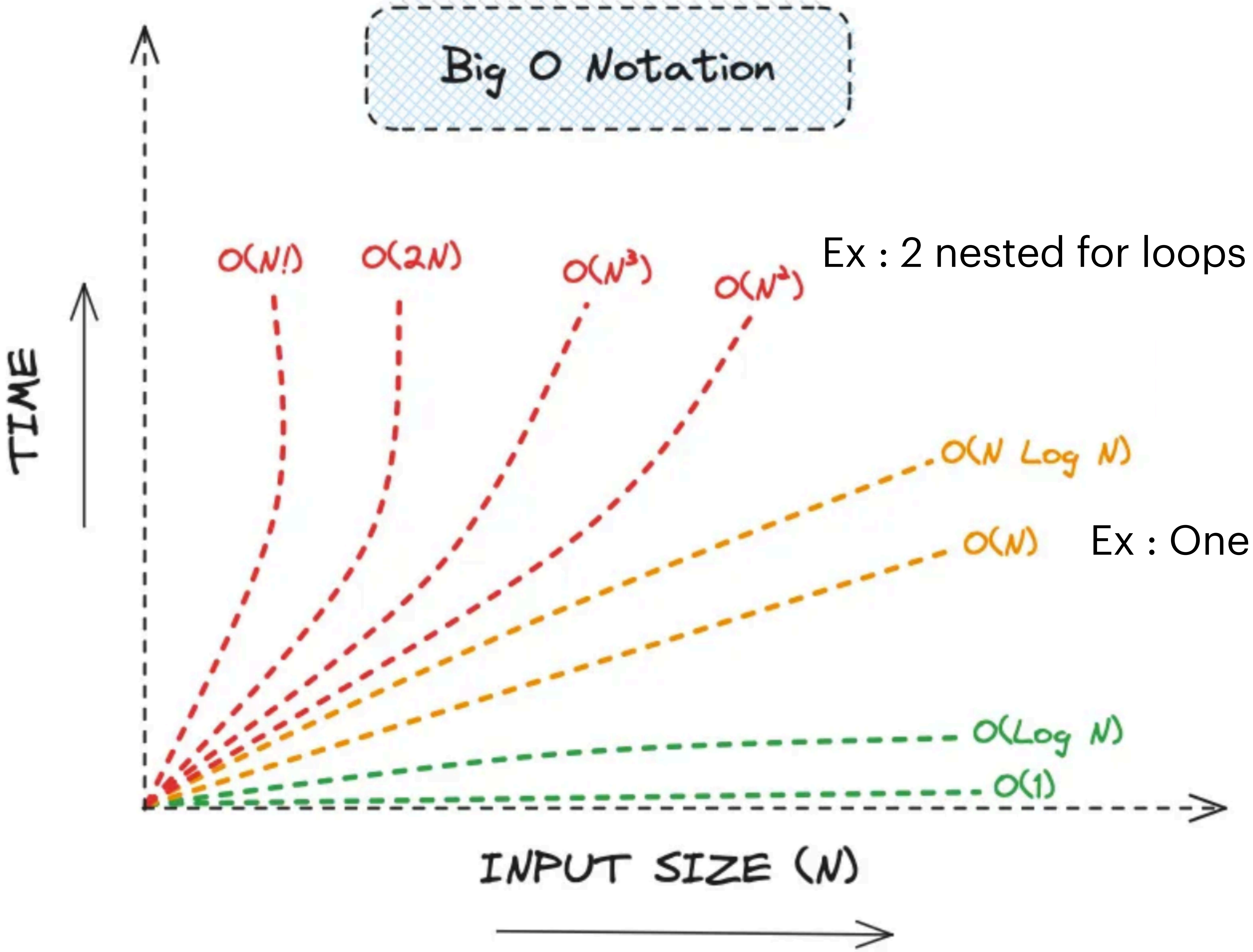
Asymptotic Notation

O-Notation



Asymptotic Notation

O-Notation



Ex : 2 nested for loops

Ex : One for loop

Ex: basic calculation

Asymptotic Notation

Ω-Notation

- Describes the lower-bound of an algorithm
- To express the best-case running time
- “ O-Notation reversed “

Ω:

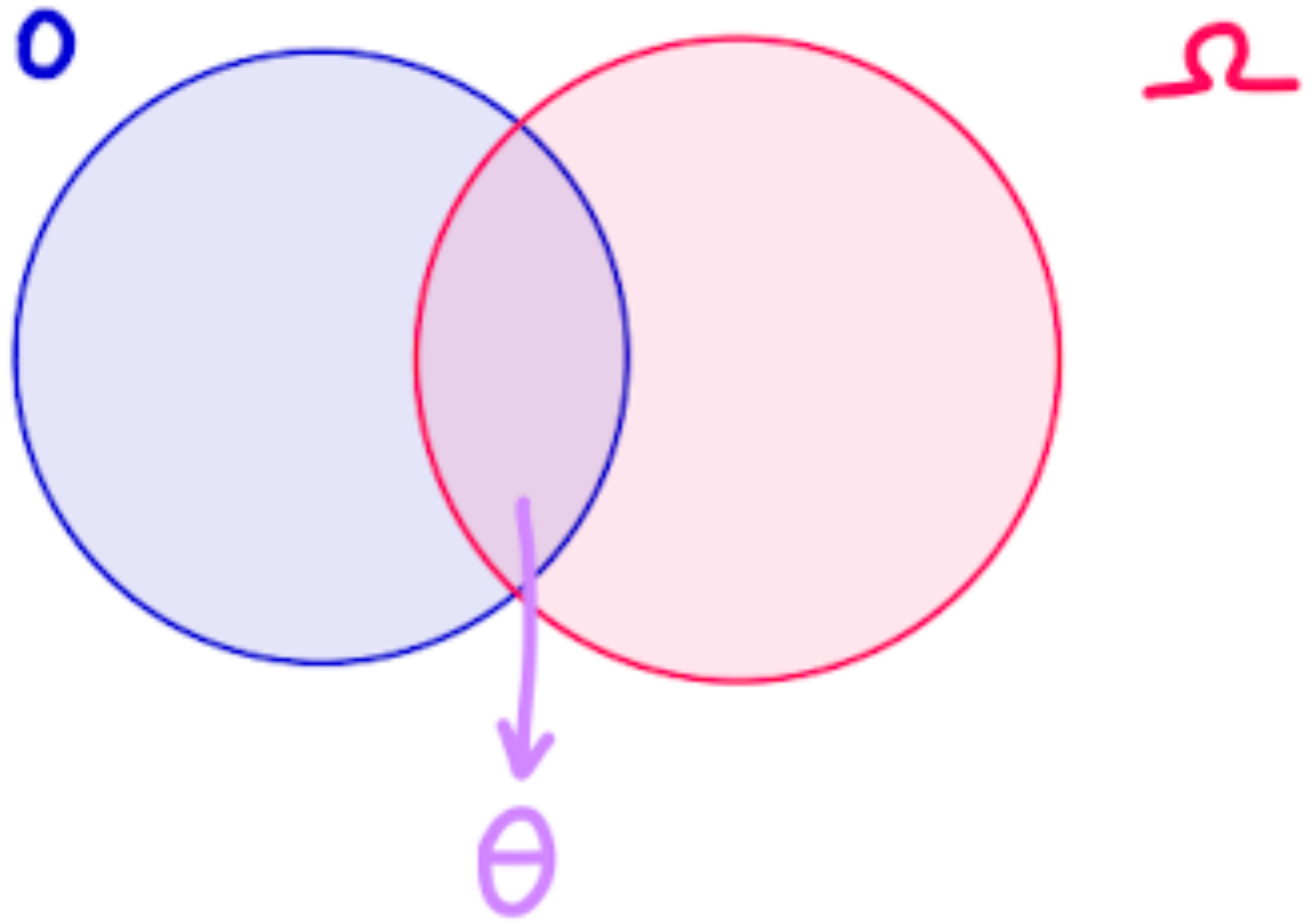
$$\sqrt{n} \leq O(n)$$

$$n \geq \Omega(\sqrt{n})$$

- n represents the amount of data that we’re passing in
- We don’t care about the differences, when those differences are constant values. We care about n

Asymptotic Notation

θ-Notation



Asymptotic Notation

Definitions

Definition 1 (O-Notation). For $f : N \to \mathbb{R}^+$,

$$O(f) := \{g : N \to \mathbb{R}^+ \mid \exists C > 0 \, \forall n \in N \; g(n) \leq C \cdot f(n)\}.$$

$$\mathcal{O}(n^2) = \{n, log(n), \sqrt{n}, log^2(n), 1, 100n^2, \cdots\}$$

Asymptotic Notation
Definitions

Definition 1 (O-Notation)**.** For $f : N \to \mathbb{R}^+$,

$$O(f) \coloneqq \{g : N \to \mathbb{R}^+ \mid \exists C > 0\, \forall n \in N\ g(n) \leq C \cdot f(n)\}.$$

Definition 1 (Ω-Notation)**.** For $f : N \to \mathbb{R}^+$,

$$\Omega(f) \coloneqq \{g : N \to \mathbb{R}^+ \mid f \leq O(g)\}.$$

We write $g \geq \Omega(f)$ instead of $g \in \Omega(f)$.

Definition 2 (Θ-Notation)**.** For $f : N \to \mathbb{R}^+$,

$$\Theta(f) \coloneqq \{g : N \to \mathbb{R}^+ \mid g \leq O(f) \text{ and } f \leq O(g)\}.$$

We write $g = \Theta(f)$ instead of $g \in \Theta(f)$.

In other words, for two functions $f, g : N \to \mathbb{R}^+$ we have

$$g \geq \Omega(f) \Leftrightarrow f \leq O(g)$$

and

$$g = \Theta(f) \Leftrightarrow g \leq O(f) \text{ and } f \leq O(g).$$

Asymptotic Notation
Definitions

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Theorem 1 (Theorem 1.1 from the script). *Let N be an infinite subset of $\mathbb{N}$ and $f : N \to \mathbb{R}^+$ and $g : N \to \mathbb{R}^+$.*

- *If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$, then $f \leq O(g)$, but $f \neq \Theta(g)$.*
- *If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C \in \mathbb{R}^+$, then $f = \Theta(g)$.*
- *If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, then $f \geq \Omega(g)$, but $f \neq \Theta(g)$.*

Asymptotic Notation

Theorem

Theorem 1 (Theorem 1.1 from the script). *Let N be an infinite subset of $\mathbb{N}$ and $f : N \to \mathbb{R}^+$ and $g : N \to \mathbb{R}^+$.*

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- *If* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$, *then* $f \geq \Omega(g)$, *but* $f \neq \Theta(g)$.

Asymptotic Notation

Mini cheat-sheet

$\lim_{n \to \infty}$: $1 < \log(\log(n)) < \log(n) < \sqrt{n} < n < n \cdot \log(n) < n \cdot \sqrt{n} < n^2 < 2^n < n! < n^n$

$n^x < x^n$ (x being fixed)

Sums

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \cdots n^2 = \frac{n(n+1)(2n+1)}{6}$$

Geometric series : $$\sum_{k=0}^{n} q^k = \frac{q^{n+1} - 1}{q - 1}$$

$$\sum_{k=0}^{3} 3^k = 3^0 + 3^1 + 3^2 + 3^3 = \frac{3^4 - 1}{3 - 1} = 40$$

Factorial

$$\frac{n}{2}^{\frac{n}{2}} \leq n! \leq n^n$$

Θ : $n \cdot \ln n = \Theta(\ln(n!))$

$\Rightarrow n \cdot \ln n \leq O(n!) \wedge n \cdot \ln n \geq \Omega(n!)$

Asymptotic Notation

Exam question

FS20

/ 4 P	a) *Landau notation:* Fill out the quiz about asymptotic notation below. You get 1P for a correct answer, -1P for a wrong answer, 0P for a missing answer. You get at least 0 points in total.

claim	true	false
$(2n + n^2 + 3)^2 = \Theta(n^4)$	☐	☐
$\frac{n}{\log n} \leq \mathcal{O}(\sqrt{n})$	☐	☐
$\log(n!) = \Theta(n \log n)$	☐	☐
$\sum_{i=1}^{\log_5 n} 5^i \geq \Omega(n \log n)$	☐	☐

Let’s take a break

Asymptotic Notation
Exam Tipps

- Try to gain an intuition
- Mind the time ! (Spend 1 min on each exercise at max)
- Don’t forget the sub results of exercise sheets

- Our “cheat sheet” to the rescue !

- Remember the definitions

Exercise Sheet 1

(Non Bonus)

Gruppen	12
Abgegeben	12

Mini Exam

Induction + Asymptotic Notation

Questions

Feedbacks , Recommendations

Nil Ozer